

The “downward continuation” of the gravity field

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Given a continuous boundary value (e.g. gravitational potential on the boundary) on a simply closed surface which includes the whole Earth, how to determine the Earth's external gravity field attracts a lot of geodesists' attention. Conventionally, to solve this problem will encounter two difficulties: 1) in the line of solving this problem in a strict sense, one might solve Molodensky problem or Bjerhammar problem^[1], both of which are very complicated in one aspect, and have not applied in solving the present problem in another aspect, because in the present case the boundary surface is not the Earth's surface but a more general surface such as the surface formulated by the flying satellites; 2) in the line of approximation approach, one could apply the truncated spherical harmonic expansion series, which might not be valid in the region near the Earth's surface^[2-4]. In both cases, one will encounter the “downward continuation” problem. As a fact, it is well known that the “downward continuation” problem is an “improperly posed” problem, that is to say, the solution is not unique, or the solution is greatly influenced by the variation of the boundary value. Theoretically, to strictly determine the gravity field, the spherical harmonic expansion approach could not be used, because the boundary surface might not be a spherical surface.

Based on the “fictitious compress recovery” approach^[5,6], the “downward continuation” problem could be satisfactorily solved. Choose a fictitious sphere K located in the interior of Earth, compress the given boundary value on the surface of the fictitious sphere, by using Poisson integral one gets the first-order regular harmonic field in the region outside the fictitious sphere. Then, one gets the difference between the original value and the first-order value on the given boundary. Repeating the above procedure, and finally one gets a fictitious field expressed by a regular harmonic series defined in the region outside the fictitious sphere, which coincides exactly with the real field to be determined in the whole region outside Earth^[5,6], naturally and satisfactorily solving the “downward continuation” problem, which might be referred as the “fictitious downward continuation” approach, with which the accuracy of the gravity field determination is in the same level as the accuracy of the given boundary value^[7].

Key words: fictitious compress recovery; “downward continuation” problem; “fictitious downward continuation”; gravity field determination

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