Error Propagation in Variational Data Assimilation

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The problem of variational data assimilation for a nonlinear evolution model is formulated as an optimal control problem to find unknown model parameters such as initial and/or boundary conditions, right-hand sides (forcing), and distributed coefficients. A necessary optimality condition reduces the problem to the optimality system (see, e.g., [1]) which includes input errors (background and observation errors); hence the error in the optimal solution. The error in the optimal solution can be derived through the errors in the input data using the Hessian of the cost functional of an auxiliary data assimilation problem. For a deterministic case it was done in [2]. In [3], a similar result was obtained for the continuous operator formulation, where a nonlinear evolution problem with an unknown initial condition was considered, when errors in the input data are random and subjected to the normal distribution. In [4], we presented an extension of the results reported in [3] for the case of other model parameters (boundary conditions, coefficients, etc.) and show that in a nonlinear case the a posteriori covariance operator of the optimal solution error can be approximated by the inverse Hessian of the auxiliary data assimilation problem based on the tangent linear model constraints. Here we investigate cases of highly non-linear dynamics, when the inverse Hessian does not properly approximate the analysis error covariance matrix, the latest being computed by the fully non-linear ensemble method with a significant ensemble size. A modification of this method that allows us to obtain sensible approximation of the covariance with a much smaller ensemble size is presented. We discuss a relationship between the performance of the incremental variational DA in strongly nonlinear cases and the proximity of the inverse Hessian and the analysis error covariance matrix. Numerical examples are presented for a nonlinear convection-diffusion problem and Burgers equation.

References

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